Reg. No.: | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.
(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
III	PART - III	CORE - 7	P23MA307	COMPLEX ANALYSIS

Date : 05.11.2024 / FN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	Which of the following is true? a) Differentiability does not implies continuity b) Differentiability implies continuity c) Continuity implies differentiability d) There is no relation between continuity and differentiability
CO1	K2	2.	A function which is analytic everywhere in a complex plane is known as a) Harmonic function b) differentiable function c) regular function d) entire function
CO2	K1	3.	The length of the circle with equation $z = z(t) = \alpha + \rho e^{it}$ $0 \le t \le 2\pi$ is ____. a) 2π b) $2\pi i$ c) $2\pi\rho$ d) $\pi\rho$
CO2	K2	4.	An arc z = z(t) is rectifiable if and only if the real and imaginary parts of z(t) are of ____________. a) bounded variation b) unbounded variation c) length of curve d) rectifiable
CO3	K1	5.	Identify $n(-\gamma, a) =$ _______________ . a) $n(\gamma, a)$ b) $-n(\gamma, a)$ c) $n(\gamma, -a)$ d) $n(-\gamma, -a)$
CO3	K2	6.	The converse of Cauchy- integral theorem is__________. a) Euler's theorem b) Liouville's theorem c) Morera's theorem d) Goursat's theorem
CO4	K1	7.	A chain is a ___________if it can be represented as a sum of closed curves. a) cycle b) length c) arc d) line
CO4	K2	8.	The integral of an exact differential over any cycle is______. a) zero c) two b) one d) three
CO5	K1	9.	Which one of the following are Laplace equation _________. a) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$ b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ c) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 1$ d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$
CO5	K2	10.	The sum of two harmonic functions and a constant multiple of a harmonic function are again __________. a) harmonic b) conjugate harmonic c) homology d) meromorphic

Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer ALL Questions choosing either (a) or (b)
CO1	K2	11a.	Elaborate the function $u = x^2 - y^2$ is conjugate harmonic . **(OR)**
CO1	K2	11b.	If $\sum_0^\infty a_n$ converges, then $f(z) = \sum_0^\infty a_n z^n$ tends to f(1) as z approaches 1 in such a way that $\lvert 1-z \rvert / (1 - \lvert z \rvert)$

CO2	K2	12a.	If f(z) is analytic in an open disk Δ then $\int_{\gamma} f(z)dz = 0$ for every closed curve γ in Δ **(OR)**
CO2	K2	12b.	Let γ be a differentiable curve given by z=z(t) $(a \le t \le b)$ .Let f(z) is a continuous function on γ .then $\left\| \int_{\gamma} f\, dz \right\| \le \int_{\gamma} \| f \| . \|dz\|$.
CO3	K3	13a.	State and prove of Lioville's theorem. **(OR)**
CO3	K3	13b.	Prove the Cauchy integral formula.
CO4	K3	14a.	A region n is simply connected if and only if $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω. **(OR)**
CO4	K3	14b.	If f(z) is analytic and not equal to zero in a simply connected region Ω, then it is possible to define single-valued analytic branches of log f(z) and $\sqrt[n]{f(x)}$ in Ω.
CO5	K4	15a.	Let f(z) be analytic except for isolated singularities a_j in a region Ω $\frac{1}{2\pi i}\int_{\gamma} f(z)dz = \sum_j n(\gamma, a_j) \operatorname{Res}_{z=a_j} f(z)$ for any cycle γ which is homologous to zero in Ω and does not pass through any of the points a_j **(OR)**
CO5	K4	15b.	If f(z) is meromorphic function in the region Ω with zero's a_j and the poles b_k then $\frac{1}{2\pi i}\int \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$ for every cycle γ which is homologous to zero in Q and does not pass through any of the zeros or poles.

Course Outcome	Bloom's K-level	Q. No	**SECTION – C (5 X 8 = 40 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K4	16a.	Analyse the C-R equation with detail explanation. **(OR)**
CO1	K4	16b.	Examine the state and prove of Lucas theorem.
CO2	K5	17a.	Express the Cauchy theorem for a rectangle . **(OR)**
CO2	K5	17b.	Prove that the line integral $\int_{\gamma} p\ dx + q\ dy$ defend in Ω,depends only on the end points of γ if and only if there exists a function U(x,y) in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$
CO3	K5	18a.	State and establish the Cauchy higher derivatives formula **(OR)**
CO3	K5	18b.	Establish the tailor series expansion .
CO4	K5	19a.	State and prove Cauchy theorem. **(OR)**
CO4	K5	19b.	If p dx+q dy is locally exact in Ω then $\int_{\gamma} p\ dx + q\ dy = 0$ for every cycle $\gamma \sim 0$ in Ω.
CO5	K6	20a.	If u is harmonic in Ω then f(z)= $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ is analytic **(OR)**
CO5	K6	20b.	Analyse the Mean value property